this phenomenon has nothing to do with the rheological complexities of the ambient liquids, and its interpretation must be sought in the different behavior of bubbles and drops in liquids. The larger the bubble is, the larger the contact area between the concave face of the leading and the convex face of the trailing bubble cap. Thus, the area of the film to be drained increases with the increasing basal diameter of the bubble. On the other hand, drops in the region of diameters between 0.004 and 0.007 m remain essentially spherical.

CONCLUSIONS

The elasticity of polymer solutions has a tendency to increase both the collision and the coalescence times of gas bubbles and liquid drops. Thus, polymer additives with strong elastic component may reduce the overall coalescence rate considerably.

NOTATION

A = material parameter, Equation (3)
b = material parameter, Equation (3)
d = basal diameter of a bubble/drop
F = axial thrust between the cone and the plate of the R18 rheogoniometer
g = acceleration due to gravity
K = material parameter, Equation (3)
n = material parameter, Equation (3)
R = radius of the cone and plate setup
r = radial distance
t_1 = collision time
t_2 = coalescence time
T = torque between the cone and the plate of the R18 rheogoniometer
U = bubble rise velocity
V = bubble volume
w = wake velocity
X_o = initial separation of bubbles/drops
X = axial distance

Greek Letters

α = complementary apex angle between the cone and the plate in the R18 rheogoniometer
$\dot{\gamma}$ = shear rate
ω = angular velocity
η = apparent viscosity
ρ = density
λ = characteristic time of a viscoelastic liquid
σ_1 = primary normal stress difference

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An Electroconductivity Technique for the Measurement of Axial Variation of Holdups in Three-Phase Fluidized Beds

JOHN M. BEGOVICH
and
J. S. WATSON

Oak Ridge National Laboratory
Post Office Box X
Oak Ridge, Tennessee 37830

A three-phase fluidized bed consists of solid phase particles fluidized by a gas and liquid flow. Although many schemes for contacting the three phases are possible, a common method is to fluidize the solid phase by the upward concurrent flow of gas and liquid. The liquid forms the continuous phase, while the gas and solids are dis-

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Correspondence concerning this note should be addressed to J. M. Begovich.

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continuous phases. This type of system may be applied to the hydrogenation of liquid petroleum fractions, the hydrogenation of unsaturated fats, coal conversion processes, and biological reactors.

Although three-phase, fluidized-bed reactors have important present and future applications, especially in hydrocarbon and coal processing, accurate design of such reactors is complicated by such factors as axial variation in reactor properties, particularly distribution of the solid catalyst. There are no data or equations for predicting reactor performance under high fluid flow rates, where these axial variations are important. Many previous investigators have used the following equations to determine the volume fraction of each phase (holdups) in a three-phase fluidized bed:

$$\epsilon_L + \epsilon_G + \epsilon_S = 1 \qquad (1)$$

$$\frac{dP}{dh} = g(\rho_L\epsilon_L + \rho_G\epsilon_G + \rho_S\epsilon_S) \qquad (2)$$

$$\epsilon_S = M_S/\rho_S A H \qquad (3)$$

where the bed height in Equation (3) is obtained either visually or from the measured pressure gradient as suggested by Kim et al. (1972) and Bhatia and Epstein (1974). Neither method is satisfactory at high fluid flow rates, because the indistinct bed height makes visual measurements extremely subjective, and the measured pressure gradient yields a bed height based on an unrealistic homogeneous bed.

ELECTROCONDUCTIVITY TECHNIQUE

To provide an alternative equation to Equation (3), the method of Achwal and Stepanek (1975, 1976) has been modified for application to a three-phase fluidized bed. As they point out, the electroconductivity of a liquid system with a fixed ion concentration at constant conditions is proportional to the cross-sectional area of the conducting liquid and inversely proportional to the length of the path between two electrodes. Thus, if the tortuosity factor remains approximately constant, the conductivity should be proportional to the liquid holdup in the bed:

$$\epsilon_L = \gamma/\gamma_o \qquad (4)$$

Similar procedures based on the same principle have been reported by several investigators. Turner (1976) measured the conductivity of liquid fluidized beds of conducting and nonconducting solids. For the nonconducting solids, he found that the data followed the Maxwell (1881) equation, simplified for the case of nonconducting particles:

$$\gamma/\gamma_o = (1 - \epsilon_S)/(1 + \epsilon_S/2) \qquad (5)$$

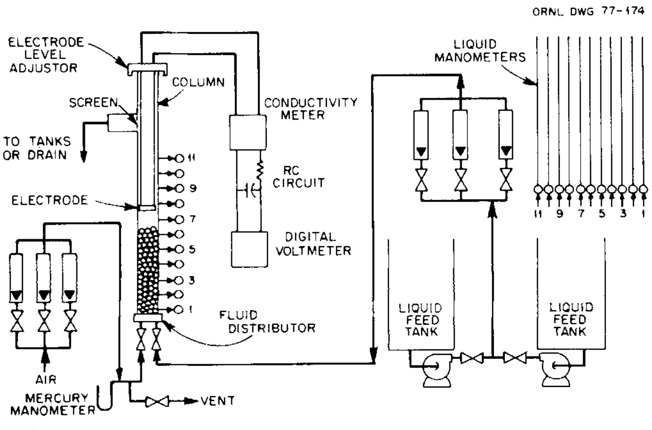

Fig. 1. Three-phase fluidization and electroconductivity apparatus.

For two-phase fluidization, Equation (5) can be presented in terms of the liquid holdup as

$$\epsilon_L = (3\,\gamma/\gamma_o)/(2 + \gamma/\gamma_o) \qquad (6)$$

Other investigators have proposed equations which yield results similar to those given by the Maxwell equation, and these are reviewed by Meredith and Tobias (1962).

Buyevich (1974) and Francl and Kingery (1954) measured the thermal conductivity of granular beds, and through the mathematical analogy between thermal and electrical conductivity they arrived at Equation (4). Our own measurements on two- and three-phase fluidized beds, as will be shown shortly, also indicated that Equation (4) is more applicable. Thus, measurement of the conductivity and pressure at various heights in the bed and application of Equations (1), (2), and (4) to incremental sections of the bed allows each of the three-phase holdups to be calculated as functions of height in the bed. This technique does not require a distinct bed height and thus allows the study of systems with high fluid flow rates which are of particular commercial interest.

EXPERIMENTAL

The experimental apparatus used to test the electroconductivity technique for measurement of axial variation in the holdups is shown in Figure 1. Various solids were fluidized by air and water in either a 7.62 cm ID or a 15.2 cm ID Plexiglas column. The two fluids were metered and then introduced through a distributor at the bottom of the bed. The air was vented to the atmosphere while the water exited through a glass tee and returned to the feed tanks. A wire-mesh screen across the glass tee prevented solids from flowing out of the column. A series of liquid manometers located at 9 cm intervals along the column wall permitted measurement of the pressure gradient in the bed.

Two platinum electrodes, each approximately 1.4 cm^2, were attached 180 deg apart on the inside of a movable Plexiglas ring. The ring, having a radial thickness of 4.7 mm and an axial width of 19 mm, was lowered or raised by two 3.2 mm OD stainless steel tubes threaded into the ring. Insulated wires were passed through the tubing and soldered to the electrodes. These wires were then connected by a coaxial cable to a Radiometer Copenhagen Type CDM2e conductivity meter. A digital millivoltmeter and a resistor-capacitor circuit (15 s time constant) connected to the conductivity meter permitted a time averaged digital read out. Saturated potassium chloride solution was added to the water in the feed tank to allow readings on the 5 mmho scale of the conductivity meter.

Solids which have been used include 4.6 mm diam glass beads with a density of 2.24 g/cm^3, 6.2 mm diam alumina beads with a density of 1.99 g/cm^3, and 6.3 mm diam Plexiglas beads with a density of 1.17 g/cm^3.

The conductivity was first measured above the bed in the liquid alone. After the liquid and gas velocities were adjusted to their desired flow rates, the liquid manometer heights were recorded. Then the conductivity between adjacent pressure taps was also recorded. Equations (1), (2), and (4) could then be solved to yield each of the three-phase holdups as a function of position in the column.

RESULTS

Comparison of Overall Phase Holdups

By placing the electrodes in the middle of the bed and assuming that the phase holdups are constant over the

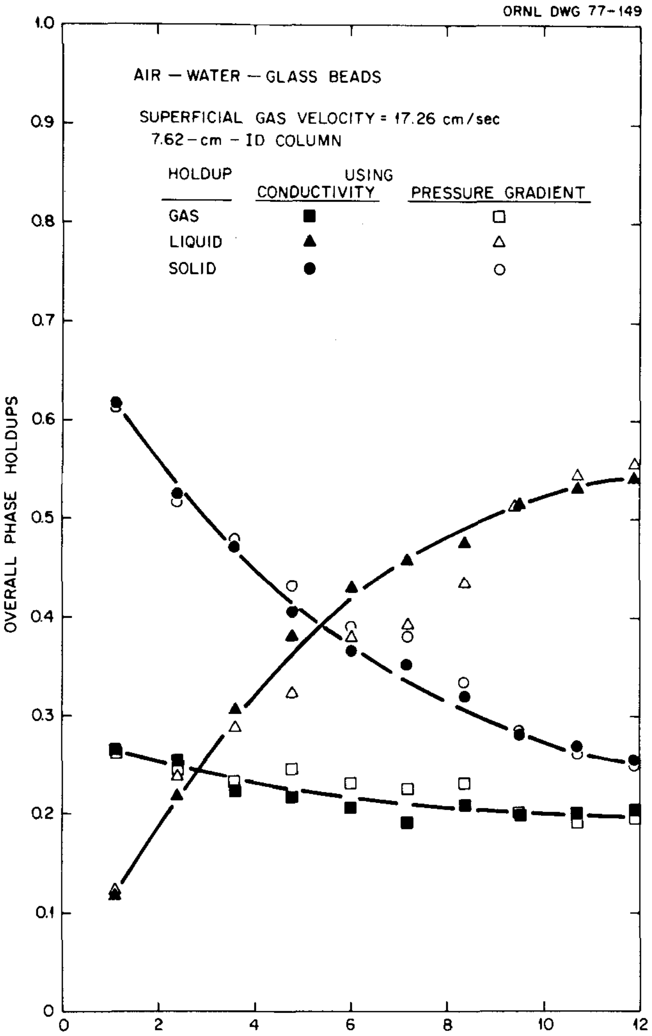

Fig. 2. Comparison of overall phase holdups calculated by the conductivity and pressure gradient methods.

entire bed, the three-phase holdups can be calculated in two ways: using Equations (1), (2), and (4) or using Equations (1), (2), and (3). Almost identical results (within 5%) are obtained with either method, as shown by a typical example in Figure 2. The glass and Plexiglas beads were used in both columns without any difficulty; however, the conductivity readings obtained using the porous alumina beads, which is the likely catalyst support for a coal liquefaction reactor, had to be corrected by a factor approximately equal to the volume fraction of liquid residing in the internal pores of the solids. Plots of the overall solid holdup using the conductivity method vs. those using the pressure gradient method result in least-squares fitted lines of nearly unity slope and zero intercept as indicated in Table 1.

TABLE 1. COMPARISON OF OVERALL SOLID HOLDUPS BY TWO DIFFERENT METHODS

$$\epsilon_{S,\text{conductivity}} = a + b\,\epsilon_{S,\text{pressure gradient}}$$

Solid	Column diameter (cm)	a	b	Correlation coefficient	Number of points
Glass	7.62	−0.052	1.067	0.985	96
Glass	15.2	−0.016	1.005	0.985	56
Plexiglas	7.62	0.005	0.862	0.914	48
Plexiglas	15.2	−0.028	0.841	0.875	84
Alumina	7.62	−0.023	1.054	0.985	98

As expected, deviations from complete agreement between the two methods occur chiefly for low values of solid holdups, that is, where the fluid flow rates are high and the bed height is not distinct. At such conditions, the pressure gradient method yields a solid holdup based on a uniform bed. The conductivity method, however, yields a solid holdup based on the middle of the bed. In reality, since the bed goes from a fairly uniform lower section through a transition region of slowly decreasing solid holdup to a region of only gas and liquid (as shown in Figure 3), the solid holdup obtained by the conductivity at the middle of the bed is lower than that obtained using the pressure gradient method.

Axial Variation in Holdups

A typical plot of the axial variation of the phase holdups is shown in Figure 3. The liquid holdup remains fairly uniform near the bottom of the bed and then increases with distance from the bottom to a constant value in the gas-liquid region above the bed. The calculated bed height (48 cm) is that obtained from the pressure gradients in and above the bed, which corresponds to that of a bed with uniform solid holdup as indicated by the horizontal dashed line.

The actual solid holdup decreases with increasing axial position in the bed, so that the observed upper limit of solids would be between 60 and 70 cm. The area under the solid holdup curve should be equal to the following modification of Equation (3):

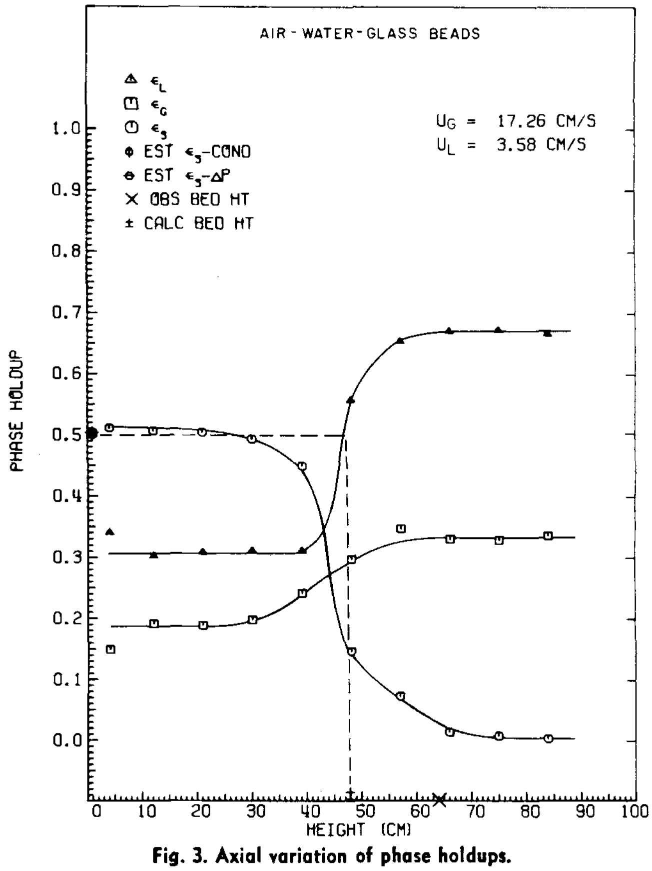

Fig. 3. Axial variation of phase holdups.

$$\int_0^H \epsilon_S dh = \frac{M_S}{\rho_S A} \qquad (7)$$

If the solid holdup is not a function of height in the bed, Equation (7) reduces to Equation (3).

The results shown in Figure 3 illustrate the shortcomings of the distinct bed height and the uniform bed assumptions used by previous investigators. In this case, which is typical of each solid used in both columns, the solid holdup slowly drops to zero from about 30 to 70 cm. This transition region is a significant fraction of the total bed height, and it must be considered in the realistic design of three-phase systems.

CONCLUSIONS

The electroconductivity technique described here can be used not only for determining the overall phase holdups in a three-phase fluidized bed, but, more importantly, it can also be used for determining the local holdups as a function of height in the column.

One disadvantage of the technique is that it can only be applied to systems with electroconductive liquids. However, since most real or prototype systems use either water or can be simulated with a fluid that can readily be made electroconductive, this handicap does not seem to be too severe.

The technique has been applied successfully to a number of systems, including porous alumina beads, if a correction is made for their internal porosity. It has shown the existence of the transition region as the bed goes from a three-phase to a two-phase system. Further work should result in correlations for the distribution of the three phases throughout the entire column. These predictive equations will help in the rational design of reactors in which local conditions throughout the bed must be considered.

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NOTATION

A = cross-sectional area of the bed
g = gravitational acceleration
h = position down the column
H = expanded bed height
M_S = mass of solids
P = local pressure in column
γ = conductivity in the bed
γ_o = conductivity in liquid alone
ϵ = phase holdup
ρ = phase density

Subscripts

G = gas phase
L = liquid phase
S = solid phase

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Application of a Self-Consistent Model to the Permeability of a Fixed Swarm of Permeable Spheres

P. M. ADLER,
P. M. MILLS,
and
D. C. QUEMADA

LBHP—(tour 33:34, 2è étage)
Université PARIS VII—2 Place Jussieu
75221—PARIS CEDEX 05
(France)

Cell models have been widely used for calculating the properties of concentrated suspensions, particularly the permeability of a porous medium composed of spherical particles. A cell consists of a particle surrounded by fluid; the models differ in the conditions imposed on the outer boundary of the cell (a full description is given

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